

Effective Interest Rate (EIR)

$$\text{EIR} = (1 + \text{Periodic Interest Rate})^{\text{number of periods in a year}} - 1$$

$$\text{EIR} = (1+i)^n - 1$$

The term "Periodic" in "Periodic Interest Rate" refers to the time between installments (daily, weekly, bi-weekly, monthly, etc.). This is important to determine the "n" in the above formula: the number of periods in a year. For example, a loan product with weekly installments would have an "n" of 52 since there are 52 weeks in a year. A loan with monthly payments would have an "n" of 12. CAUTION: so-called "monthly" payments may actually be payments every 28 days. (As in, "your payment falls due every fourth Thursday.") Whereas there are 12 months in a year, there are 13 periods of 28 days. This will affect the EIR calculation.

Note the difference between the EIR and the easier-to-calculate Annual Percentage Rate (APR), the interest rate which, for example, all U.S. lenders must publish as per the Truth in Lending law:

APR = the Periodic Interest Rate times the number of periods in a year

$$\text{APR} = i * n$$

Examples:

Loan Amount	Frequency of Installments	Number of Installments	Periodic Interest Rate	Annual Percentage Rate	Effective Interest Rate
2,500	Weekly	8	1%	52%	67.77%
7,500	Weekly	8	1%	52%	67.77%
7,500	Weekly	12	1%	52%	67.77%
Not relevant	Bi-Weekly	Not relevant	1%	26%	29.53%
" "	Monthly	" "	1%	12%	12.68%
" "	Every 4 weeks	" "	1%	13%	13.81%

The examples in the first three rows demonstrate that the loan amount and the actual number of installments to be paid are not relevant to determine interest rates. What *is* relevant is the periodic interest rate and the number of such periods per year.

There's only one important detail left to cover: how, exactly, to determine that all-important periodic interest rate...

PERIODIC INTEREST RATE

The financial term "interest rate" refers to the number applied to the outstanding balance of a loan to determine the amount of interest to be paid with each installment. Below are a few examples of a periodic interest rate being applied to a loan:

Possibility 1: Decreasing installments.

In this example, the amount of principal to be paid on each installment remains constant. Assume a loan of 1,000 to be paid in four monthly installments. The amount of principal to be paid in each installment = $1,000 \div 4 = 250$.

Assume a periodic (in this case, monthly) interest rate of 1%. The corresponding amortization table is as follows:

Total Installment	Principal	Interest	Outstanding Balance
			1,000.00
260.00	250.00	10.00	750.00
257.50	250.00	7.50	500.00
255.00	250.00	5.00	250.00
252.50	<u>250.00</u>	<u>2.50</u>	0.00
	1,000.00	25.00	

Note how the value of each total installment (first column) declines in value, due to the lower interest to be paid on the declining outstanding balance.

Possibility 2: Equal installments.

Many micro-enterprise programs prefer equal installments. The relevant formula for this next discussion is as follows:

$$\text{Loan Amount} \times \frac{i (1+i)^n}{(1+i)^n - 1} = \text{Size of Each Installment}$$

This is the formula that is programmed into financial calculators, as well as certain functions in spreadsheet software. As before, "i" is the periodic interest rate, but this time the "n" refers to the number of installments in the loan. (One would think that the original creators of these formulae could have chosen another letter to avoid confusion....)

When three of the variables in the above formula are known, the fourth can be calculated. For example, if we know the loan amount, the interest rate we want to charge and the number of installments that the client will have to pay, this formula determines the value of each installment. Using the same numbers from the previous example (loan amount = 1,000; number of installments = 4; periodic interest rate = 1%), we can determine the size of each installment using even a normal calculator: 256.28. The respective amortization table is as follows:

Total Installment	Principal	Interest	Outstanding Balance
			1,000.00
256.28	246.28	10.00	753.72
256.28	248.74	7.54	504.98
256.28	251.23	5.05	253.75
256.28	<u>253.75</u>	<u>2.53</u>	0.00
	1,000.00	25.12	

The formula determines the constant value of the installments. To determine how much principal must be paid in *each* installment, calculate first the interest due by multiplying the periodic interest rate (1%) by the outstanding balance. Then, the Principal = Total Installment - Interest. Note that although the periodic interest rate is the same, the monetary amount of total interest actually paid increases from 25.00 to 25.12. This is due to the extra liquidity enjoyed by the borrower by having lower payments during the first half of the repayment schedule (eg. 256.28 versus 260 for the first installment). Finally, be careful of the final payment: due to rounding, total principal may vary slightly from the total amount disbursed. For example, 1% of 253.75 = 2.54 instead of the 2.53 shown above. Using 2.54 for interest would cause principal in the final installment to be 253.74. Total principal paid for the four installments, therefore, would be 999.99 and not 1,000. Since this is the stuff of nightmares for accountants everywhere, the principal and interest to be paid in the final installment is adjusted, if necessary, to ensure that everything adds up the way it should.

OK, everything so far still seems to be fairly straightforward. Challenges occur, however, when programs introduce "flat interest rates" and commissions:

Possibility 3: Flat Interest Rates and their APR and EIR equivalence.

"Flat interest rates" are not interest rates per se but rather a simple percentage used often within NGO marketing strategies to disguise the true (ie. higher) interest rate of a microenterprise loan:

Loan Amount	Term	"Interest"	Total Debt	Each Installment
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1,000	4 monthly installments	1% "Flat" per month = 4% flat	1,000 + (1,000 times 4%) = 1,040	1,040) 4 = 260.00
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In this case, the loan amount, number of installments and value of each installment are known. Finding the "i" in the following formula is best done using either a financial calculator or the appropriate functions in your favorite spreadsheet software:

$$\text{Loan Amount} \times \frac{i (1+i)^n}{(1+i)^n - 1} = \text{Size of Each Installment}$$

Financial calculators make short work of this calculation. These machines have four crucial buttons:

- "PV" (present value) = Loan Amount (to be entered as a negative number: -1,000)
- "PMT" (payment) = Size of Each Installment (260.00)
- "n" (number of payments to be made: 4)
- "i" (the elusive periodic interest rate).

On a Hewlett-Packard HP-12C, all you have to do is enter the three known numbers, then press the "i" button. After a few seconds, the display gives the result. In this case: 1.58749908...% This is the periodic interest rate. Since the payments are monthly, the APR = 19.05% and the EIR = 20.80%.

CAUTION: if you have a different financial calculator, work with it until you get the same results as those shown above. Some financial calculators assume that all loans must have multi-year periods and calculate, by default, a yearly "i". Not good. Give it away to a friend and buy yourself another one.

Alternately, a spreadsheet can be utilized. The final page of this handout portrays an approach for setting up a spreadsheet for determining APRs and effective interest rates. The amortization table, using the periodic interest rate on the declining balance to separate principal from interest in each installment, proves the accuracy of the "i" calculation:

Total Installment	Principal	Interest	Outstanding Balance
			1,000.00
260.00	244.13	15.87	755.87
260.00	248.00	12.00	507.87
260.00	251.94	8.06	255.93
260.00	<u>255.93</u>	<u>4.07</u>	0.00
	1,000.00	40.00	

Possibility 4: Commissions

OK, we've got a 1% "flat" interest rate with four monthly installments (APR equivalent = 19.05%). Let's add a commission of, say, 5% to spicen things up. This fee will be subtracted from the amount to be disbursed. For our 1,000 loan, the commission will equal $1,000 * 5\% = 50.00$.

In effect, the net amount being loaned to the client is 1,000 less the 50.00 commission. Therefore, the PMT key on the calculator must reflect the new loan amount. With "PV" equal to -950, "n" as 4, and "PMT" as 260.00, the "i" comes out to 3.7215..%. The APR, therefore, is 44.66% and the EIR is 55.03%!!

To make such charges sound more palatable, programs may "finance at 0% interest" the commission to be charged, meaning that the 50.00 fee will be distributed equally among the four installments to be paid. Therefore, the total value of each installment will increase to $260.00 + (50.00 / 4) = 272.50$. What's the new periodic interest rate?

"PV" is back to -1,000, the "n" stays at 4, and the "PMT" rises to 272.50. The "i" now comes out to 3.5385..% (APR = 42.46%; EIR = 51.78%). Note that the "i" declines slightly because it is taking the program longer to get its money: instead of retaining all of it up front, it must wait until the installment is paid each month.

Possibility 4

NGOs, banks, credit unions, credit card companies, car salespeople, etc., are very clever at elevating portfolio yields. In addition to commissions, any number of additional ploys can be used to disguise the actual APR. A good suggestion is to follow the flow of money, figure out what the net "PV" and "PMT" are and work it out. Develop an amortization table with your results for independent verification. Finance is slippery, so proceed with caution.

An example from the real world: how to turn 18.90% into 40.2%!

1. 4 days into the VISA billing period, a traveler obtained \$100 from a convenient ATM at Heathrow airport. The exchange rate for the day was 0.618046, meaning that the US\$ equivalent was 161.80
2. VISA calculated the "Average Cash Advance" for the 30-day period: 140.22. This was calculated by multiplying the 161.80 advance by the 26 days that the advance was outstanding, and dividing the result by the 30 days in the billing period: $26 \text{ days} * 161.80 / 30 \text{ days} = 140.22$. (Upon reflection, the traveler was quite surprised to see that VISA did not charge 140.23 ...) This was multiplied by the 18.9% APR to obtain the finance charge of \$2.18.
3. a 2% flat charge (\$3.24) was added to the total cash advance:

Cash Advances	Daily Periodic Rate	Corresponding APR	Finance Charge	ANNUAL PERCENTAGE RATE
\$140.22	.05178%	18.90%	\$2.18	
Cash Advance	Charge per Transaction			
\$161.80	2.0%, Minimum \$2.00		\$3.24	40.2%

$$PV = -161.80; \text{ PMT} = 161.80 + 2.18 + 3.24 = 167.22; \text{ n} = 1; \text{ i} = 3.349815... * 12 \text{ months} = 40.2\%$$

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EXCEL Spreadsheet Model for Calculating Interest Rates

Note: italicized terms at right are range names and refer to the cell in column A found on the same line. For example, the range name for cell A1 is LOAN. The use of range names (optional) simplifies the formulas found in cells A6 and A11.

	A		
1	\$1,000.00	Loan Amount	<i>LOAN</i>
2	10	Number of payments in loan agreement	<i>TERM</i>
3	52	Number of payment periods in a year (eg: weekly installments = 52)	
4	24.0%	Nominal Annual Interest Rate	
5	0.46%	Nominal Periodic Interest Rate (A4 / A3)	<i>INTEREST</i>
6	\$102.56	Value of Each Payment:	A6: =PMT(<i>INTEREST</i> , <i>TERM</i> , <i>LOAN</i> *-1)
7			
8	0.0%	Commission deducted from Loan Amount at time of Disbursement	<i>COMMISSION</i>
9	\$0.00	Charge added to each Installment	<i>FEE</i>
10			
11	0.46%	Periodic Interest Rate	A11: =RATE(<i>TERM</i> , <i>PAYMENT</i> + <i>FEE</i> , <i>LOAN</i> *-1*(1- <i>COMMISSION</i>))
12	24.0%	Annual Percentage Rate (APR)	A11 * A3
13	27.1%	Effective Annual Interest Rate	(1 + A11) ^ A3 - 1

The two significant formulas are located in cells A6 and A11. For programs using a "flat" interest rate to determine payments, replace the formula in A6 with the corresponding payment.

For Lotus users:

the formula for cell A6 is as follows: @PAYMT(*LOAN*,*INTEREST*,*TERM*)

the formula for cell A11 is: @IRATE(*TERM*,*PAYMENT*+*FEE*,*LOAN**(1-*COMMISSION*))

For earlier versions of LOTUS, replace PAYMT for the cell A6 formula, with PMT